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Modeling a Low-Energy Ballistic Lunar Transfer Using Dynamical Systems Theory

Jeffrey S. Parker* and George H. Born†
University of Colorado, Boulder, Colorado 80309

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Low-energy ballistic lunar transfers are analyzed, modeled, and constructed in this paper using dynamical systems theory. An example ballistic lunar transfer is presented here that a spacecraft may use to transfer between a 185 km circular low Earth orbit and a halo orbit about the Earth–moon L_2 point using no deterministic maneuvers apart from the translunar injection maneuver. This transfer is modeled in the patched three-body model by mapping the invariant manifolds of libration orbits. An energy analysis is presented that shows how a spacecraft's energy with respect to the Earth and moon as well as its Jacobi constant in the sun–Earth/moon and Earth–moon three-body systems change throughout the transfer. The low-energy transfer and the methods used to produce it are then validated by reconstructing the trajectory in the Jet Propulsion Laboratory's DE405 planetary and lunar ephemeris model of the solar system. Finally, it is shown that the low-energy transfer requires approximately 19.7% less ΔV than a conventional direct lunar transfer to the same orbit. Discussions are provided throughout the paper that consider how a ballistic lunar transfer may be a beneficial trajectory option for the transport of material between the Earth and the moon.

Nomenclature

a	=	semimajor axis, km
C	=	Jacobi constant, normalized units
EL_i	=	ith Lagrange point in the sun–Earth system
F	=	orbit family parameter
L_1	=	collinear Lagrange point between the two primaries
L_2	=	collinear Lagrange point beyond the smaller primary
LL_i	=	ith Lagrange point in the Earth–moon system
m_{moon}	=	mass of the moon, kg
m_{sun}	=	mass of the sun, kg
p	=	perturbation direction flag
r_{SOI}	=	radius of the three-body sphere of influence, km
W^S	=	stable invariant manifold
W^U	=	unstable invariant manifold
ΔV	=	change in velocity, km/s
Δt_m	=	manifold propagation time, normalized time
ϵ	=	perturbation magnitude, normalized units
θ	=	sun–Earth–moon angle, deg
τ	=	revolution number about an orbit

I. Introduction

THE transportation of material to the moon has become a topic of increasing attention in recent years. Several space agencies are exploring low-cost alternative methods, rather than conventional methods, to transfer spacecraft and cargo to orbits about the moon. The purpose of this paper is to demonstrate how to use dynamical systems theory to model, analyze, and construct low-energy ballistic transfers between the Earth and the moon.

The techniques presented in this paper readily produce low-energy transfers between the Earth and lunar halo orbits or other unstable Earth–moon three-body orbits. With a trivial addition that is described in this paper, these trajectories may be extended to transfer material from the Earth to low lunar orbits or to the surface of the moon.

Conventional transfers to lunar orbits, including transfers to low lunar orbits and lunar halo orbits, require two large maneuvers and fewer than six days of transfer time [1,2]. These transfers may be approximated by Hohmann transfers [3]. By extending the transfer's duration by several months and using the sun's gravity, one can substantially reduce or even eliminate the need for the second large maneuver. In every practical case studied, these longer transfers require less energy than conventional transfers. These transfers are therefore called low-energy ballistic lunar transfers (BLTs) [1].

BLTs have been constructed before: the Japanese Hiten spacecraft implemented a low-energy transfer in 1991 to reach the moon [4]. Because of Hiten's success, many different strategies have been developed to build BLTs. Most of the documented strategies involve targeting schemes, designed to adjust some set of maneuver parameters to build the BLT [5–7]. These documented schemes certainly produce practical lunar transfers, but they normally require a priori knowledge of the characteristics of the transfer to do so [5,8,9]. In addition, none of the strategies to date have been applied to the problem of transferring a spacecraft to a lunar halo orbit.

Dynamical systems theory has been used to demonstrate the existence of low-energy lunar transfers with only basic knowledge of the near-Earth dynamical system [10]. Previously, the only dynamical systems studies related to the construction of BLTs have been limited to the planar problem [10,11]. This paper extends that work and demonstrates how to use dynamical systems theory to build practical BLTs in three dimensions.

Dynamical systems theory provides many tools that are useful to understand how a spacecraft takes advantage of the gravity of the Earth, moon, and sun to perform a low-energy BLT. This paper studies the energy and Jacobi constant of a spacecraft with respect to the primary bodies to explain why the dynamics of the four-body problem yield low-energy lunar transfers. Furthermore, dynamical systems theory provides the means to parameterize BLTs such that full families of similar BLTs may be effectively explored. In this way, mission designers may quickly evaluate many types of BLTs for their potential use in practical spacecraft missions.

This paper is organized in the following manner. Section II discusses the background of low-energy ballistic lunar transfers, dynamical systems theory, and the models used in this paper. Section III introduces the different segments of a BLT, qualitatively

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*Colorado Center for Astrodynamics Research, 431 UCB; parkerjs@gmail.com. Member AIAA.

†Director, Colorado Center for Astrodynamics Research, 431 UCB, Fellow AIAA.

describing a BLT from different perspectives. Section IV discusses how to model a BLT using dynamical systems theory. It is shown that dynamical systems tools allow a mission designer to compartmentalize the design process in the same way that patched-conic methods compartmentalize each segment of a conventional interplanetary transfer. That is, one may analyze and design each segment of a BLT independently. Section V provides an analysis of an example BLT from an energy perspective. This analysis gives insight into how a spacecraft ballistically changes its energy with respect to the Earth and moon. Section VI discusses how to construct BLTs in the patched three-body model using dynamical systems theory. Section VII validates the example BLT by transferring it from the patched three-body model into the JPL DE405 model of the solar system. Section VIII compares the performance of the BLT presented in this paper with conventional transfers between the same orbits. Finally, Sec. IX summarizes the conclusions that may be drawn from this paper.

II. Background

A. Previous Work Building Low-Energy Lunar Transfers

Low-energy lunar transfers have been constructed in the past using a variety of strategies. As early as 1968, Conley [10] began using dynamical systems methods to construct such transfers, although the transfers he constructed were restricted to two dimensions. Conley's method involved constructing a trajectory that transferred from the Earth's vicinity to the moon's vicinity through the neck region about one of the collinear libration points in the Earth–moon system.

In the late 1980s and early 1990s, Belbruno [5] began developing a method to construct lunar transfers using his weak stability boundary theory [4,8,9]. The method involves targeting the region of space that is in gravitational balance between the sun, Earth, and moon, without involving any three-body periodic orbits or the invariant manifolds of such orbits. Ballistic capture occurs when the spacecraft's two-body energy becomes negative, as illustrated by Yamakawa et al. [12]. Ivashkin [13–15] developed a very similar targeting method in the early 2000s.

In the mid-1990s, other methods were developed to construct a lunar transfer that take advantage of the chaos in the Earth–moon three-body system [6,16]. Transfers have been produced using these methods that require less energy than conventional direct transfers, but they require between nine months and several years of transfer time.

In 2000, Koon et al. [11,17] constructed a planar lunar transfer that was almost entirely ballistic using the techniques involved in Conley's method. Similar to Conley's method, Koon et al. constructed a transfer by targeting a trajectory within the interior of the stable invariant manifold of a planar libration orbit about the Earth–moon L_2 point. Once inside the interior of the stable manifold, the spacecraft ballistically arrives at some temporarily captured orbit about the moon.

Dynamical systems theory has been shown to be particularly useful for constructing BLTs in the planar problem because the problem is very well-defined. The four-dimensional phase space of the planar circular restricted three-body problem (CRTBP) is reduced to three dimensions by specifying a particular Jacobi constant. Koon et al. [11] showed that the two-dimensional invariant manifolds of a Lyapunov orbit act as a separatrix, dividing the three-dimensional energy surface into interior and exterior regions. A spacecraft in the exterior region of the Earth–moon system could then target a trajectory within the interior region of the energy surface to transfer through the neck of the forbidden region into the interior of the system. Unfortunately, the algorithms presented by Koon et al. cannot be trivially moved into the three-dimensional CRTBP system. The spatial CRTBP has a phase space with six dimensions; by specifying a particular Jacobi constant, the system is reduced to a five-dimensional energy surface. Even the three-dimensional manifolds of quasi-periodic Lissajous orbits [18] cannot clearly divide a five-dimensional energy surface into well-defined interior and exterior regions. Hence, the methods developed by Conley [10] and Koon et al. [11] fail to produce the same kind of success in three dimensions as they demonstrate in two dimensions; new techniques must be developed to solve the three-dimensional transfer problem.

The present study demonstrates how to use dynamical systems theory to construct three-dimensional transfers from the Earth to unstable three-body orbits in the Earth–moon system. If needed, one may use a given Earth–moon three-body orbit as a staging orbit en route to a target lunar orbit or even the surface of the moon. Thus, the techniques presented in this paper may be used to build an end-to-end transfer from the Earth to any type of orbit at the moon.

B. Dynamics in the Sun–Earth–Moon System

The circular restricted three-body problem may be used as a model to describe the motion of a massless particle in the presence of two massive bodies: for example, the motion of a spacecraft in the presence of two stellar or planetary bodies [19]. The motion of a spacecraft in the vicinity of the Earth may be modeled by the sun–Earth/moon three-body model (referred to here as the *sun–Earth* model, in which the sun is the primary body and the Earth–moon barycenter is the secondary body) or by the Earth–moon three-body model, depending on the location of the spacecraft. The three-body models constructed here use solar system constants provided by Vallado [20].

The CRTBP permits the existence of many periodic orbit solutions [1]. Halo orbits are very interesting and well-known periodic solutions to the CRTBP. Many papers in the literature provide more information about the existence and construction of such libration orbits [21–23].

Many authors have studied how to take advantage of lunar halo orbits for practical missions to the moon [24–26]. Halo orbits are of particular use for lunar communication and navigation satellites [2,27]. For visualization, Fig. 1 shows a plot of example halo orbits about the lunar L_1 and L_2 points. The halo orbits are viewed in the Earth–moon synodic reference frame: namely, the reference frame that corotates with the motion of the Earth and moon about their barycenter [19]. This coordinate frame's x axis extends from the Earth–moon barycenter toward the moon, its z axis extends from the barycenter toward the direction of positive angular momentum, and its y axis completes the right-hand coordinate frame. Because the force field in the CRTBP is symmetric about the x–y plane, and because halo orbits are asymmetric about this plane, each halo-orbit solution to the CRTBP comes in a symmetric pair. The two varieties of halo orbits are known as northern and southern orbits [22]. As one can see in Fig. 1, a satellite in a southern orbit spends more than half of its time below the x–y plane, which gives that satellite benefits for communicating with objects in the southern hemisphere of the moon.

Halo orbits do not have a constant set of orbital elements like conic orbits. However, halo orbits exist in well-defined families, a characteristic that allows a particular halo orbit in a family to be uniquely specified by a single parameter [28]. In this paper, halo orbits are identified by the value of their Jacobi constant [19].

Because halo orbits in the Earth–moon system are unstable, they have associated invariant manifolds [29–31]. If a spacecraft is perturbed while traveling along an unstable halo orbit, the spacecraft will exponentially fall away from the orbit, tracing out a single trajectory that shadows the orbit's unstable manifold. Similarly, a spacecraft with the right state will traverse the orbit's stable manifold and asymptotically approach that halo orbit. Figure 2 shows plots of the stable manifolds of example L_1 and L_2 halo orbits. One can see that there is an *interior* and an *exterior* half of each manifold. A spacecraft that travels along any one of the trajectories shown will asymptotically arrive on the host halo orbit. In this paper, a spacecraft that has come within 100 km of a halo orbit is considered to be on the halo orbit.

Although the periodic halo orbits only truly exist in the simplified three-body problem, trajectories do exist in the real solar system that shadow many of these orbits.[‡] Fig. 3 shows two sets of plots of an

[‡]Certain orbits, however, may be significantly affected or destroyed when they are transferred into the real solar system, due to resonances with the sun [32].

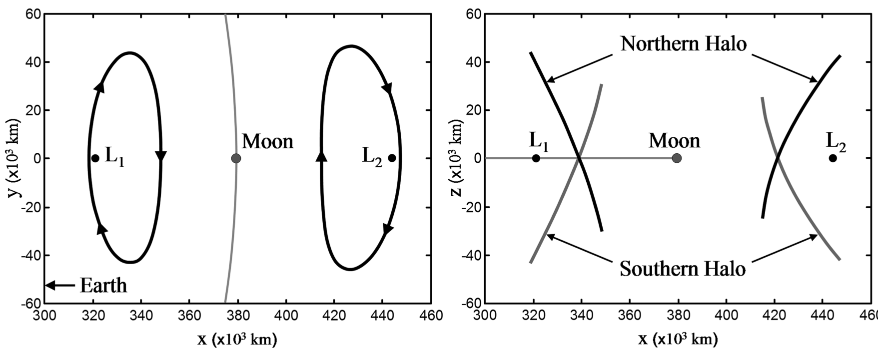

Fig. 1 Two perspectives of example halo orbits about the lunar L_1 and L_2 points: viewed from above (left) and viewed from the side (right) in the Earth–moon synodic reference frame.

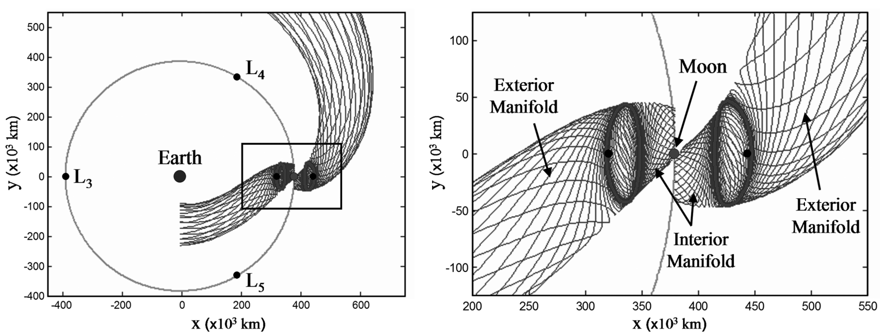

Fig. 2 Plots of the stable invariant manifolds of example L_1 and L_2 halo orbits in the Earth–moon system. A spacecraft that travels along any one of these trajectories will asymptotically arrive on the corresponding halo orbit.

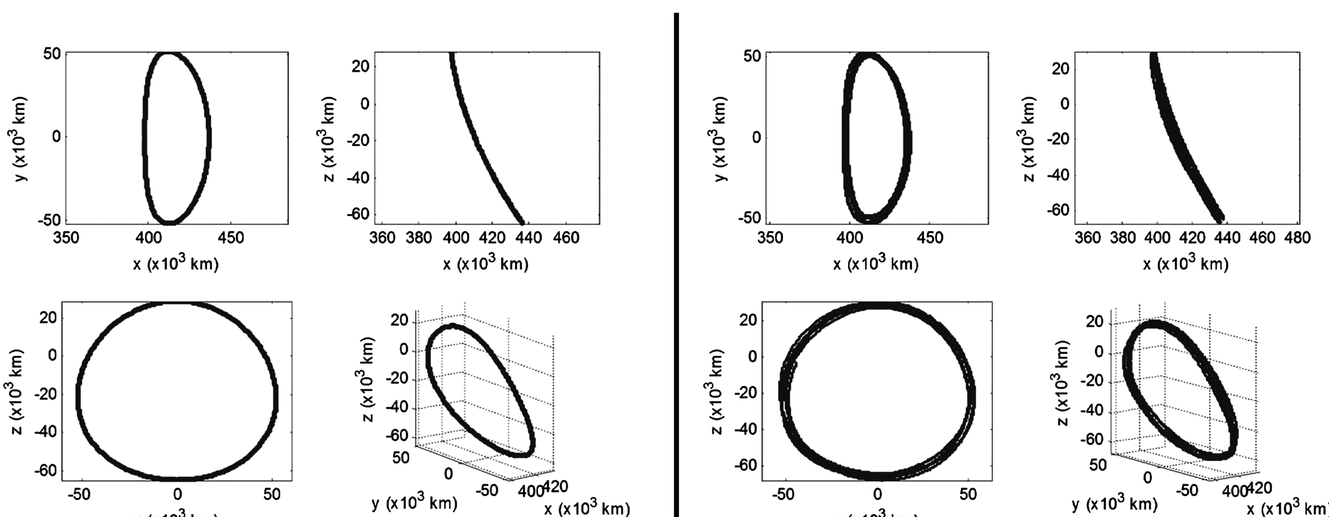

Fig. 3 Plots of an example lunar L_2 halo orbit produced in the CRTBP system (left) and in the full JPL DE405 ephemeris system (right). The orbits are viewed from four different perspectives each.

example halo orbit that exists about the moon's L_2 point: one set produced in the CRTBP and the second set produced in the full JPL DE405 ephemeris system. One notices that the trajectory produced in the ephemeris system is not perfectly periodic, but is well approximated by the CRTBP halo orbit. Similarly, the invariant manifolds of real orbits shadow the invariant manifolds of orbits produced in the CRTBP.

C. Patched Three-Body Model

The patched three-body model uses purely three-body dynamics to model the motion of a spacecraft in the sun–Earth–moon–spacecraft four-body system. When the spacecraft is near the moon, the spacecraft's motion is modeled by the Earth–moon three-body system. Otherwise, the spacecraft's motion is modeled by the sun–Earth three-body system. For simplicity it is assumed that the Earth–moon system is coplanar with the sun–Earth system. The boundary of these two systems is referred to as the three-body sphere of influence (3BSOI); it is analogous to the two-body sphere of influence used in the patched conic method of interplanetary mission design. The patched three-body model retains many of the desirable characteristics of the CRTBP, while permitting a spacecraft in the near-Earth environment to be affected by all three massive bodies, albeit only two massive bodies at any given moment.

The Earth–moon 3BSOI is defined in this paper to be the boundary of a sphere centered at the moon with a radius r_{SOI} computed using the following relationship:

$$r_{\mathrm{SOI}} = a\left(\frac{m_{\mathrm{moon}}}{m_{\mathrm{sun}}}\right)^{2/5} \tag{1}$$

where m_{moon} and m_{sun} are the masses of the moon and sun, respectively, and a is the average distance between the sun and moon, equal to approximately 1 AU. Thus, the three-body sphere of influence has a radius of approximately 159,200 km. A sphere of that radius centered at the moon includes the moon's L_1 and L_2 points, but not the other three Lagrange points. This is satisfactory for the purposes of this study, and is consistent with other studies [12], but should be further studied for missions that require use of the other three Lagrange points. This model will be validated in Sec. VII.

III. Mission Segments of a BLT

This section qualitatively describes the motion of a spacecraft following a low-energy ballistic lunar transfer. Figure 4 shows two example BLTs, viewed in the sun–Earth rotating frame from above the ecliptic. The main aspects of a BLT may be summarized by discussing the low-Earth-orbit (LEO) parking orbit, the outbound trajectory, the encounter with the moon, and then the options that a spacecraft has once it arrives at its lunar orbit.

A. LEO Parking Orbit

Ballistic lunar transfers may begin in any LEO parking orbit, including orbits with an inclination compatible with a launch from Cape Canaveral, provided that the spacecraft injects onto the outbound trajectory at the right time. It has been found that BLTs may be constructed that depart the Earth during most days of each month and during any month of the year [1,33]. Some departure dates correspond to BLTs with shorter transfer durations or to BLTs that require less energy than others. These preliminary results include only the purely ballistic transfers produced in this research [1,33]; it is very likely that with the addition of one or two small maneuvers, BLTs may be constructed that depart the Earth during any day of the year.

B. Outbound Trajectory

After the spacecraft performs the injection maneuver from its LEO parking orbit, the spacecraft travels well beyond the orbit of the moon and into the region of space that is substantially influenced by the sun's gravity as well as the Earth's. It is beneficial to describe the spacecraft's motion from both two-body and three-body perspectives, because both perspectives help to paint the full picture.

From a two-body perspective, the spacecraft begins by transferring from its LEO orbit onto a highly eccentric orbit about the Earth: an orbit with an apogee far beyond the moon's orbital radius. As the spacecraft approaches and traverses the apogee of this orbit, the spacecraft lingers long enough to give the sun a large amount of time to perturb its orbit. During this time, the sun's gravity effectively raises the perigee of the spacecraft's orbit. By the time the spacecraft has returned to its orbital perigee, the spacecraft's orbit will have changed so much that its perigee is now near the radius of the moon's orbit about the Earth. As the spacecraft approaches this new perigee, it encounters the moon.

From a three-body perspective, the spacecraft begins by transferring from its LEO orbit onto a trajectory that shadows the stable manifold of a sun–Earth three-body orbit (typically a quasi-periodic libration orbit). The spacecraft approaches this orbit as it approaches the Earth's L_1 or L_2 point, but the spacecraft does not enter the orbit. The spacecraft then ballistically transfers to a trajectory that shadows the three-body orbit's unstable manifold. This trajectory takes the spacecraft to the lunar encounter. The observed transfers occur without any required deterministic maneuvers, due to the unstable nature of this three-body orbit.

The outbound trajectory travels well beyond the orbital radius of the moon. With some clever planning, the outbound trajectory may therefore be designed to pass by the moon on its way out. By performing a lunar flyby during the outbound trajectory, the launch costs may be further reduced.

During this transfer, the spacecraft requires stationkeeping to remain on its proper trajectory. The stationkeeping cost is minimal and may be accounted for by trajectory correction maneuvers; the Genesis spacecraft followed a similar low-energy transfer and required only approximately 8.87 m/s of ΔV per year [34–36].

C. Lunar Encounter

As the spacecraft approaches the moon, it arrives on the stable manifold of an Earth–moon three-body orbit, such as a lunar halo orbit. As the spacecraft follows this stable manifold, it asymptotically approaches the three-body orbit; hence, the transfer requires no final orbit-insertion maneuver to inject into the lunar three-body orbit.

The three-body orbit may be planar or three-dimensional; it may orbit the moon's L_1 point, its L_2 point, or the moon itself. One of the only requirements is that the orbit must be unstable; otherwise, it will not have useful invariant manifolds and an orbit-insertion maneuver will be required. The orbit is typically chosen to either meet mission requirements (such as the requirements of a communication satellite at L_1 or L_2) or to be used as a staging orbit before transferring to a final lunar orbit.

D. Available Options from the Lunar Three-Body Orbit

Once a spacecraft has arrived at the lunar three-body orbit, the spacecraft has several options. First of all, it may remain there for as long as desired, or at least until its stationkeeping fuel budget is exhausted (which may be years). Lunar halo orbits may be a desirable location for communication and/or navigation satellites; they may also be a desirable location for space stations or servicing satellites.

The spacecraft may transfer from the three-body orbit to a different three-body orbit in the Earth–moon system for very little energy, provided that both orbits are unstable and have the same Jacobi constant [31,37,38]. For instance, the spacecraft might arrive at a lunar L_2 halo orbit and then later transfer to a lunar L_1 halo orbit.

The spacecraft may also transfer from the nominal three-body orbit onto its unstable manifold and follow that trajectory to a desirable stable lunar orbit. It has been found that nearly any low lunar orbit is accessible in this way, and every transfer studied has required a smaller orbit-insertion maneuver than any conventional direct transfer to the same low lunar orbit [1]. An example of such a transfer will be described in more detail subsequently.

Similarly, the spacecraft may follow the unstable manifold of the three-body orbit down to the surface of the moon. It has been found

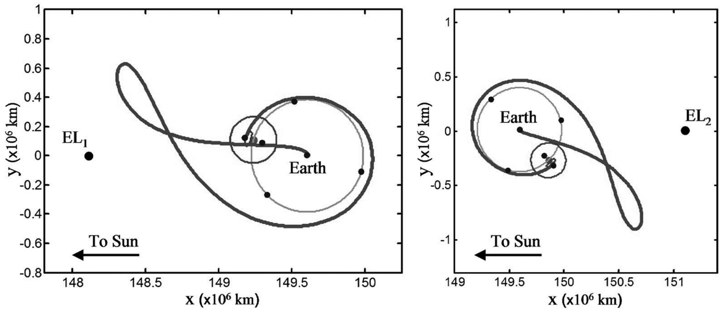

Fig. 4 Two example low-energy ballistic lunar transfers, viewed in the sun–Earth rotating frame from above the ecliptic. The 3BSOI is shown encircling the moon.

that any point on the surface of the moon may be reached, although some points require several orbits about the moon before touchdown [1,27]. Again, the required ΔV to land from the lunar three-body orbit is smaller than the required ΔV to land following a conventional direct transfer from the Earth.

Finally, the spacecraft has the option to return to the Earth following an Earth-return trajectory. Every BLT has a symmetric Earth-return counterpart; the Earth-return trajectory does not need to be the same type that has been used to arrive at the lunar orbit.

If the spacecraft's final destination is not the lunar three-body orbit, then the spacecraft does not need to inject into that orbit. Instead, the orbit's stable manifold may be used to guide the spacecraft to its final destination, rather than to inject the spacecraft onto the three-body orbit. The stable manifold may be used as an initial guess into a trajectory-optimization routine, such as a multiple-shooting differential corrector [18,39].

The discussion henceforth will graphically illustrate some example options that a spacecraft has upon arriving at a lunar halo orbit. Figure 5 shows one such lunar halo staging orbit and its unstable manifold. A spacecraft on this halo orbit may depart along any one of these trajectories. These trajectories fly by the moon at different radii and inclinations, indicating that many different final lunar orbits are accessible from this staging orbit. When one considers all halo orbits in the family of L_2 halo orbits, one finds that nearly any low lunar orbit may be accessed by a BLT. Figure 6 shows the available options that have been identified for the radius and inclination of lunar orbits that may be accessed by southern lunar L_2 halo orbits. The shaded field in the right plot has been constructed by sampling the unstable manifolds of hundreds of halo orbits and interpolating between the results. The highlighted points in the plot on the right are those points that are accessible from the example southern halo staging orbit shown in Fig. 5. Northern halo orbits can access the same set of lunar orbits, except with a negative inclination. In each case, it is assumed that the orbit-insertion maneuver is performed at the perilune of the unstable manifold, but this is not a necessary requirement.

IV. Modeling a Ballistic Lunar Transfer Using Dynamical Systems Theory

Section III qualitatively described each portion of a low-energy ballistic lunar transfer; this section describes how to use dynamical systems theory to model a BLT in the patched three-body model.

A BLT may be modeled as a series of transfers from one three-body orbit to another. After the spacecraft launches from its LEO parking orbit, the spacecraft transfers to the vicinity of a three-body orbit in the sun–Earth system, referred to in this section as the *Earth staging orbit*. The spacecraft's LEO departure trajectory follows the flow of the Earth staging orbit's stable manifold. Once in the vicinity of the Earth staging orbit, the spacecraft falls away from the staging orbit, following the flow of that orbit's unstable manifold. The trajectory is chosen so that it encounters the stable manifold of a three-body orbit in the Earth–moon system, referred to in this section as the *lunar staging orbit*. The spacecraft may use the lunar staging orbit as a final destination or as a transitory orbit, as summarized in Sec. III.D. To generalize the modeling process even further, a BLT may be modeled as a transfer from Earth to one or more Earth staging orbits to one or more lunar staging orbits and then to some final destination.

Sections IV.A and IV.B discuss viable Earth and lunar staging orbits, respectively; Sec. IV.C shows an example of a simple BLT modeled using a single Earth staging orbit and a single lunar staging orbit. An end-to-end solution has been constructed using a Lissajous orbit [18] about the sun–Earth L_2 point (EL_2) and a halo orbit about the Earth–moon L_2 point (LL_2).

A. Earth Staging Orbits

Many types of three-body orbits may be used as Earth staging orbits in the process of modeling or constructing a BLT. A proper staging orbit must meet the following requirements:

1) The orbit must be unstable.

2) If the orbit is the first Earth staging orbit, then the orbit's stable manifold must intersect LEO or the launch asymptote; otherwise, the orbit's stable manifold must intersect the preceding staging orbit's unstable manifold.

3) The orbit's unstable manifold must intersect the following staging orbit's stable manifold, be it another Earth staging orbit or a lunar staging orbit.

A quasi-periodic Lissajous orbit has been selected for the example in Sec. IV.C because it meets each of these requirements. Unfortunately, quasi-periodic orbits and their invariant manifolds are difficult to visualize because they never retrace their paths. This section illustrates the validity of a Lissajous orbit by showing that halo orbits are viable candidates to be used as Earth staging orbits.

Figure 7 shows four perspectives of the family of northern halo orbits centered about the sun–Earth L_2 point. Lissajous orbits span a very similar region of space, but often do not extend as far in the z axis.

Most libration orbits in the sun–Earth system are unstable and hence meet the preceding requirement 1. This discussion will assume that a halo orbit from the family shown in Fig. 7 will be used as the only Earth staging orbit en route to a lunar staging orbit. Figure 8 shows two plots of an example halo orbit about the sun–Earth L_2 point and the interior half of its stable manifold. One can see that this stable manifold intersects the Earth. Thus, a spacecraft may make a single maneuver to transfer from a LEO parking orbit to a trajectory on this halo orbit's stable manifold; this satisfies requirement 2 for this itinerary. Similarly, Fig. 9 shows two plots of the same halo orbit's unstable manifold, showing that trajectories exist that intersect the moon's orbit about the Earth. Thus, a spacecraft on, or sufficiently near, the halo orbit may use the orbit's unstable manifold to guide it to intersect the moon (satisfying requirement 3). The invariant manifolds of Lissajous orbits with similar Jacobi constants also demonstrate the same properties, making them viable candidates for BLT staging orbits.

B. Lunar Staging Orbits

Many different Earth–moon three-body orbits may be used as lunar staging orbits; the example BLT modeled in this section uses a halo orbit about the Earth–moon L_2 point as its lunar staging orbit because it meets all of the requirements.

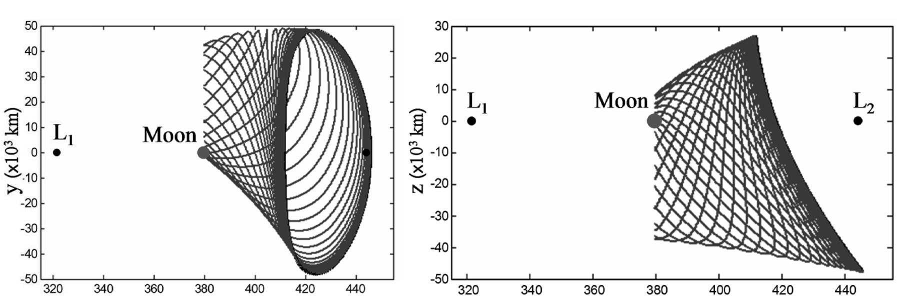

Fig. 5 An example lunar halo staging orbit and its unstable manifold, viewed in the Earth–moon rotating frame from above (left) and from the side (right). A spacecraft on this halo orbit may depart along any one of the trajectories shown.

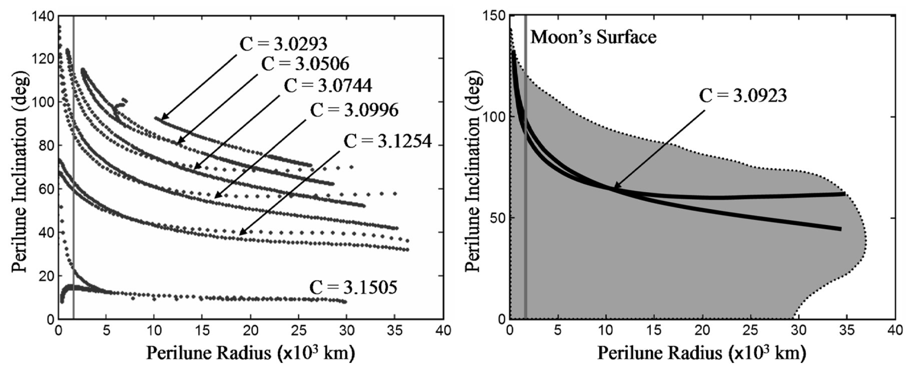

Fig. 6 Radii and inclination combinations that may be obtained at perilune of the unstable manifolds of six different lunar L_2 halo orbits (left), where each orbit's available options are labeled with that orbit's Jacobi constant, and of many orbits in the family of southern halo orbits (right). The highlighted options in the plot at right correspond to the available options for the halo orbit shown in Fig. 5.

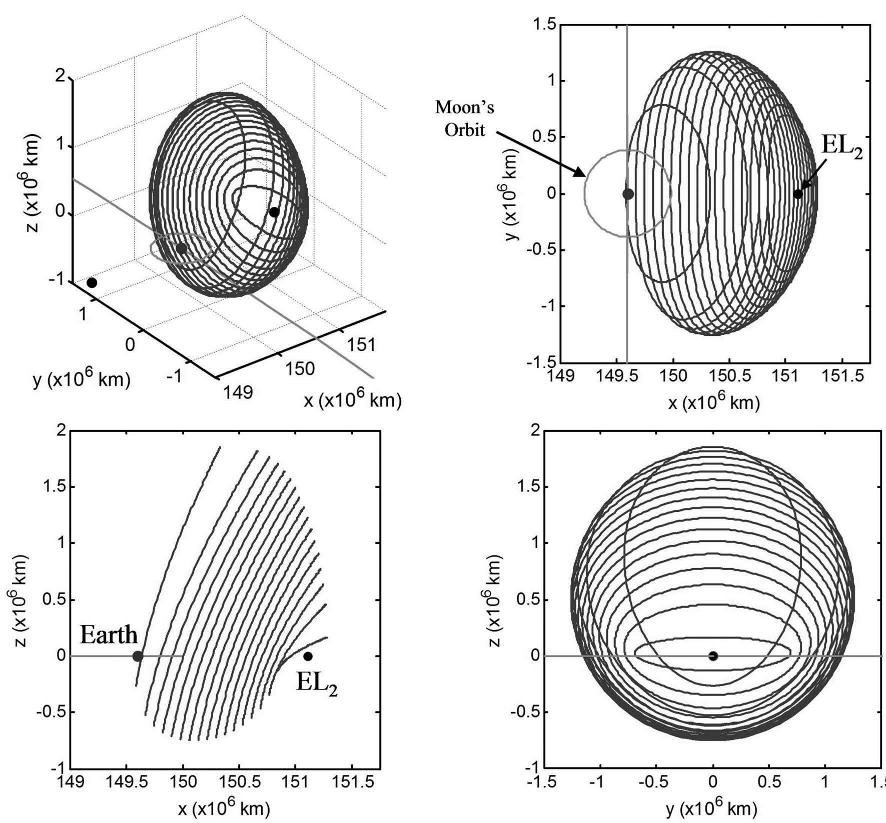

Fig. 7 Four perspectives of the family of northern halo orbits about the sun–Earth L_2 point.

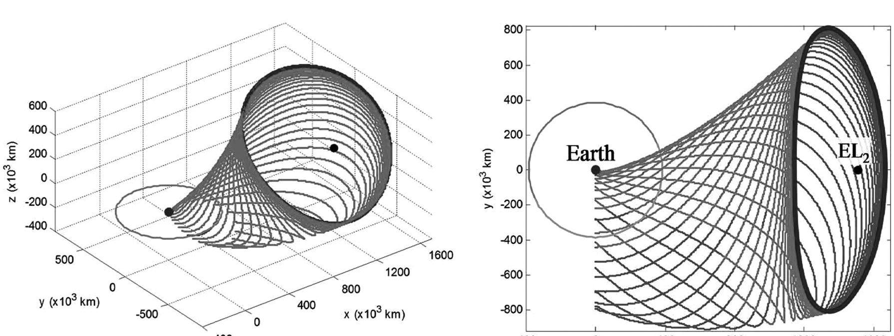

Fig. 8 Two perspectives of an example northern halo orbit about the sun–Earth L_2 point, shown with the interior half of its stable manifold. One can see that the stable manifold intersects the Earth.

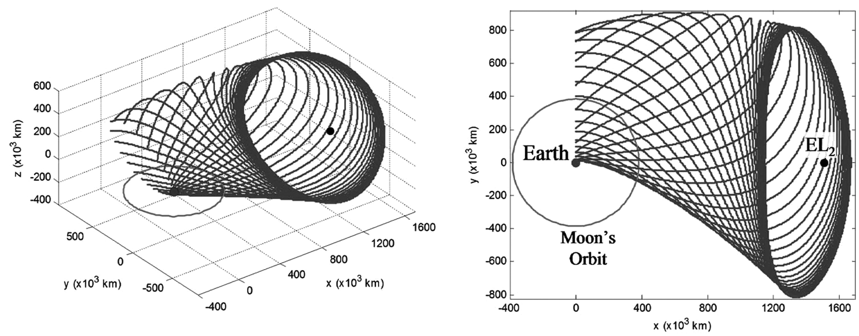

Fig. 9 Two perspectives of the same northern EL_2 halo orbit shown in Fig. 8, this time shown with the interior half of its unstable manifold. One can see that the unstable manifold intersects the moon's orbit.

The requirements for a lunar staging orbit typically come from the requirements of the mission itself, as summarized in Sec. III.D. The following list summarizes the additional requirements imposed on the lunar staging orbit:

1) The orbit must be unstable.

2) The orbit's stable manifold must intersect the unstable manifold of the preceding staging orbit, be it the previous lunar or the previous Earth staging orbit.

3) If the orbit is the final lunar staging orbit, then it must meet any requirements derived from the mission; otherwise, the orbit's unstable manifold must intersect the following lunar staging orbit's stable manifold.

There are many families of Earth–moon three-body orbits that satisfy requirement 1, including the family of lunar L_2 halo orbits. The family of halo orbits about the Earth–moon L_2 point closely resembles the family of halo orbits about the sun–Earth L_2 point shown in Fig. 7 and will not be shown here, for brevity.

Figure 10 shows two perspectives of an example LL_2 halo orbit along with its exterior stable manifold, propagated in the patched three-body model. If a spacecraft were to target a trajectory on this manifold, it would asymptotically approach and eventually arrive onto the staging orbit. Thus, if a spacecraft were able to transfer from the Earth staging orbit's unstable manifold onto this LL_2 halo orbit's stable manifold, then the spacecraft would have achieved a ballistic transfer to this lunar orbit from LEO.

C. Example of a Modeled BLT

An example BLT has been modeled using dynamical systems theory and will be presented here. It is a fairly simple example of a BLT: it consists of a single Earth staging orbit and a single lunar staging orbit. A Lissajous orbit about the sun–Earth L_2 point has been selected to be the Earth staging orbit, although it will be visualized here by a halo orbit with the same Jacobi constant. A lunar L_2 halo orbit has been selected to be the only lunar staging orbit. The BLT has been produced in the patched three-body model.

Figure 11 shows the first portion of the three-dimensional BLT in two perspectives. The spacecraft is launched from a 185 km low Earth orbit, travels outward toward the sun–Earth L_2 point along a trajectory that shadows the stable manifold of an EL_2 libration orbit, skims the periodic orbit, and then travels toward the moon. Figure 11 shows the representative halo orbit and its stable manifold $W^S_{EL_2}$; the stable manifold of the actual Lissajous staging orbit does an even better job of mapping out the flow of the spacecraft's motion in space.

Figure 12 shows two perspectives of the same transfer trajectory, but this time plotted with the Earth staging orbit's unstable manifold $W^U_{EL_2}$. One can see that as the spacecraft departs the vicinity of the Earth staging orbit and approaches the moon, its trajectory shadows the unstable manifold of the Earth staging orbit.

Figure 13 shows the same two perspectives of the three-dimensional BLT plotted alongside the lunar staging orbit's stable manifold $W^S_{LL_2}$. One can see that the BLT intersects the manifold in full phase space, indicating that the spacecraft has injected into the LL_2 halo orbit. Once in the final Earth–moon halo orbit, the spacecraft has all of the options presented in Sec. III.D available to it.

Figure 14 shows a top-down perspective of the entire three-dimensional BLT with all three manifolds displayed.

D. Discussion

The dynamical systems method of modeling, analyzing, and constructing BLTs takes advantage of three-body structures. Three-body orbits and their corresponding manifolds exist in families; a BLT constructed using three-body orbits is therefore also a member of a family of similar BLTs. It has been found that many such families exist, each with its own range of performance parameters [1,33].

V. Energy Analysis of a BLT

Ballistic lunar transfers harness the sun's gravity to reduce the ΔV requirements of a lunar transfer. It is useful to observe how the two-body energy of the spacecraft with respect to each of the massive bodies changes throughout the transfer. It is also useful to observe how the moon affects the spacecraft's sun–Earth Jacobi constant and especially how the sun affects the spacecraft's Earth–moon Jacobi constant. These energy changes are explored in this section, applied to the example BLT produced in Sec. IV. Other BLTs have been found to behave in a very similar fashion.

To begin this analysis, Fig. 15 shows plots of the distance between the spacecraft and both the Earth and moon as the spacecraft traverses

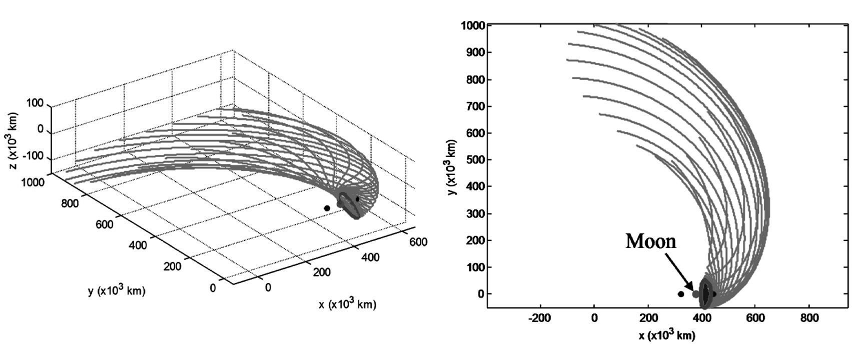

Fig. 10 Two perspectives of an example southern halo orbit about the Earth–moon L_2 point, shown with the exterior half of its stable manifold. One can see that the stable manifold quickly departs the moon's vicinity and may then intersect the unstable manifold of the Earth staging orbit.

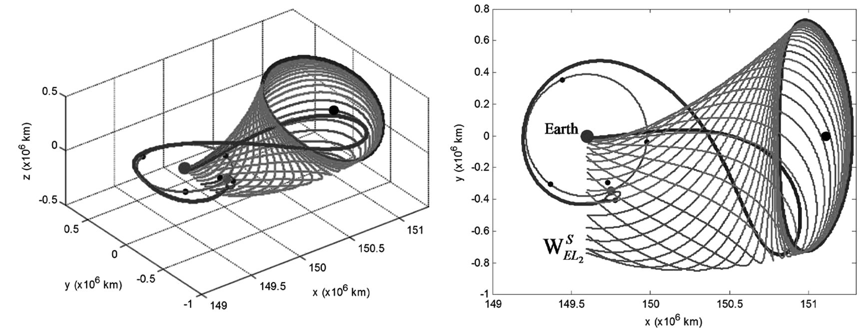

Fig. 11 Two perspectives of the first portion of the example BLT, modeled using the stable manifold of a halo orbit about the sun–Earth L_2 point. One can see that the spacecraft's outbound motion shadows the halo orbit's stable manifold.

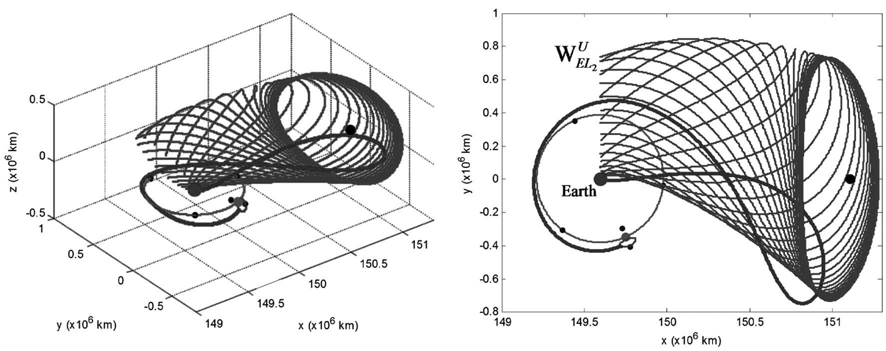

Fig. 12 Two perspectives of the second portion of the example BLT, modeled using the unstable manifold of a halo orbit about the sun–Earth L_2 point. One can see that as the spacecraft departs the vicinity of the Earth staging orbit and approaches the moon, its trajectory shadows the unstable manifold of the Earth staging orbit.

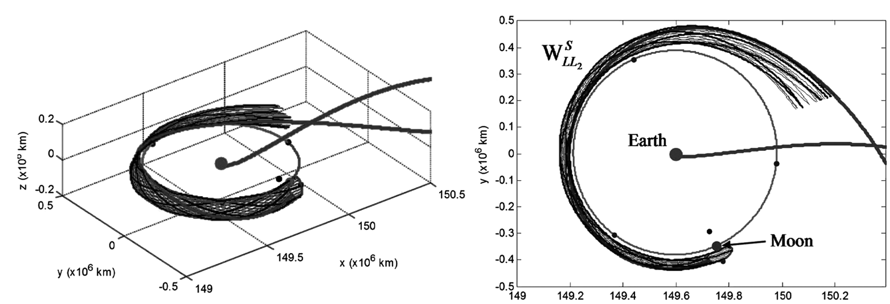

Fig. 13 Two perspectives of the third portion of the example BLT, modeled using the stable manifold of a halo orbit about the Earth–moon L_2 point. Every fourth trajectory has been darkened for visualization purposes. One can see that the BLT intersects the manifold in full phase space, indicating that the spacecraft injected into the LL_2 halo orbit.

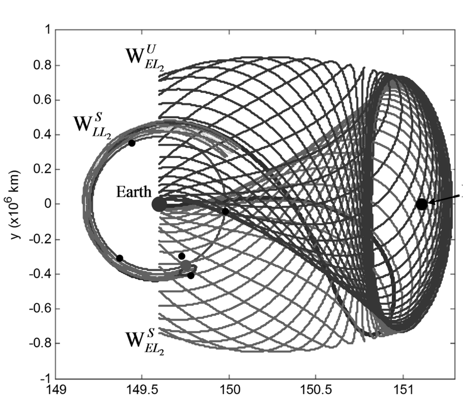

Fig. 14 Top-down perspective of the example BLT, shown with all three manifolds that have been used to model it.

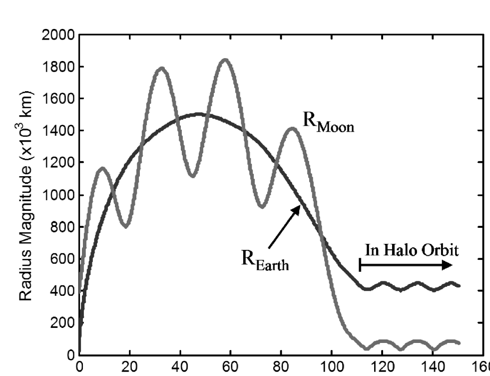

Fig. 15 Magnitude of the radius vector of the spacecraft with respect to the Earth and the moon as the spacecraft traverses the example BLT.

the BLT. This is a useful illustration because both the spacecraft's two-body energy and its Jacobi constant vary as functions of distance to these bodies. By observing Fig. 15, one can determine the time at which the spacecraft arrives at its lunar halo-orbit destination.

It is expected that the two-body energy of a spacecraft with respect to the Earth increases over time due to the sun's gravity, because the spacecraft's perigee radius gradually rises throughout the transfer. Figure 16 shows the two-body specific energy of the spacecraft with respect to the Earth throughout the BLT. One can see that the spacecraft's energy does indeed rise while it is in the vicinity of the Earth staging orbit. The energy then begins to vary wildly once it enters the lunar halo orbit, which makes sense because the halo orbit only exists in the presence of both the Earth and the moon, balancing the gravity of both bodies. Figure 17 shows four other two-body orbital elements of the spacecraft with respect to the Earth as the spacecraft traverses the BLT, including the spacecraft's semimajor axis, perigee radius, eccentricity, and ecliptic inclination. One can see that the sun's gravity increases the spacecraft's semimajor axis and perigee radius as the spacecraft traverses the Earth staging orbit. The sun's gravity reduces the spacecraft's eccentricity and inclination with respect to the Earth. The spacecraft enters the lunar halo orbit at approximately 110 days after launch, beyond which the moon's gravity is the dominant source causing each of the spacecraft's orbital elements to vary over time.

It is also expected that the spacecraft's two-body energy with respect to the moon decreases as the spacecraft approaches and ballistically inserts into the lunar halo orbit. Figure 18 shows the two-body specific energy of the spacecraft with respect to the moon throughout the BLT. One can clearly see that the spacecraft's specific energy drops as it approaches the lunar halo orbit. Furthermore, its energy drops below zero, satisfying some authors' requirements to be temporarily captured by the moon [1,12,40].

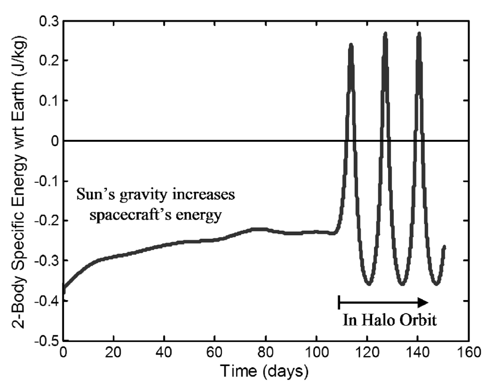

Fig. 16 Two-body specific energy of a spacecraft with respect to the Earth over time as it traverses an example BLT.

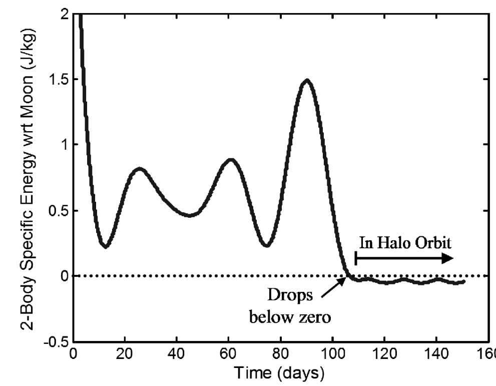

Fig. 18 Two-body specific energy of a spacecraft with respect to the moon over time as it traverses an example BLT.

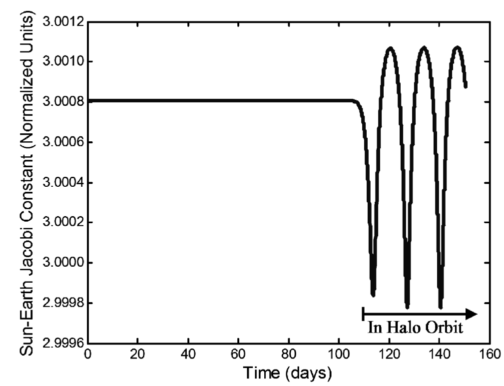

Fig. 19 Evolution of the spacecraft's Jacobi constant with respect to the sun–Earth three-body system as the spacecraft traverses the example BLT.

Figures 19 and 20 show the evolution of the spacecraft's Jacobi constant with respect to the sun–Earth and Earth–moon three-body systems, respectively, as the spacecraft traverses the example BLT. The spacecraft's trajectory has been constructed in the patched three-body model; hence, the spacecraft's Jacobi constant will be constant in one or the other three-body system at any given time, depending on which three-body system is responsible for the given segment of the spacecraft's trajectory. The spacecraft's motion has been modeled by the sun–Earth three-body system during the first 105 days of the transfer. After the spacecraft has crossed the Earth–moon 3BSOI, its motion is then modeled by the Earth–moon three-body system.

Figure 20 presents a compelling case that it is possible to build BLTs to lunar halo orbits or other unstable Earth–moon three-body

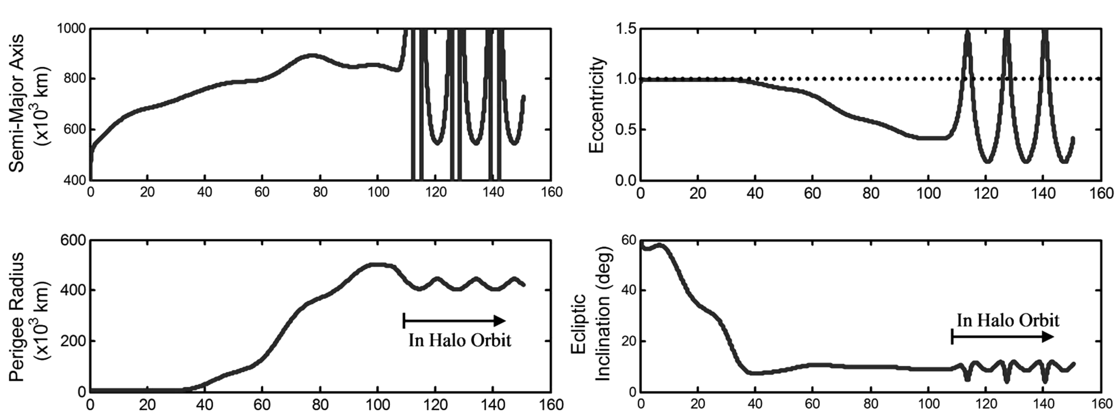

Fig. 17 Four two-body orbital elements of the spacecraft with respect to the Earth as the spacecraft traverses the example BLT, including the spacecraft's semimajor axis (top left), perigee radius (bottom left), eccentricity (top right), and ecliptic inclination (bottom right).

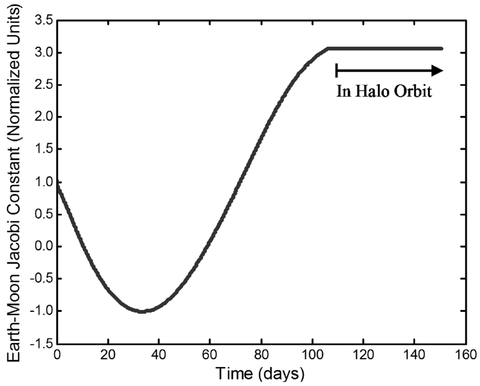

Fig. 20 Evolution of the spacecraft's Jacobi constant with respect to the Earth–moon three-body system as the spacecraft traverses the example BLT.

orbits with a wide variety of different Jacobi constants. If the spacecraft traversing the example BLT had arrived at the moon slightly earlier or slightly later, it could have transferred to a lunar halo orbit with a different Jacobi constant. Furthermore, it may be possible for a spacecraft to depart one lunar halo orbit, traverse through the sun–Earth environment for some time, and return to the moon on the stable manifold of a different lunar halo orbit. The present research found that it is indeed possible to build BLTs to lunar halo orbits within a wide range of Jacobi constants [1], but more work needs to be accomplished to determine how to take advantage of the time series shown in Fig. 20 to target a lunar halo orbit with a specified Jacobi constant.

VI. Constructing a BLT in the Patched Three-Body Model

Modeling a BLT using dynamical systems theory involves the use of several staging orbits and their corresponding invariant manifolds in the Earth–moon and sun–Earth systems. If a mission designer wishes to construct a BLT that intentionally visits certain staging orbits, then the BLT may be constructed in the same manner that it is modeled. More often, a mission designer only wishes for the spacecraft to reach the final lunar orbit, no matter its route through the sun–Earth system. In that case, the methods used to construct a BLT may be simplified.

Ballistic lunar transfers have been constructed in this paper by propagating the stable manifold of the final lunar halo orbit backward in time for a set amount of time. After each trajectory has been propagated, the perigee point of the trajectory is identified. A proper BLT may be identified as one for which the perigee point corresponds to some desired value (e.g., an altitude of 185 km). In this manner, a practical BLT may be constructed between the Earth and the lunar three-body orbit without identifying any required staging orbit.

A. Parameters

The dynamical systems method of constructing ballistic lunar transfers provides a natural set of six parameters that may be used to define each BLT. In the patched three-body model, this set may be described by the parameters F, C, θ, τ, p, and Δt_m. Each of these parameters will be described in this section.

1. Orbit Family Parameter: F

Depending on the mission requirements, one may wish to target any type of Earth–moon three-body orbit. The parameter F is a discrete variable that describes the orbit family that contains the desired target orbit. The example BLT presented in this paper had the parameter F set to describe the family of southern LL_2 halo orbits. There are certainly symbolic ways to represent each family of three-body orbits, but using text to do so provides a clear description of which family is being used.

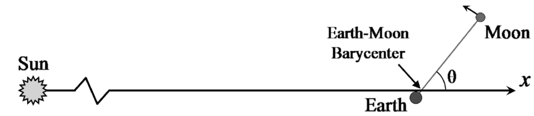

Fig. 21 Illustration of θ, the sun–Earth–moon angle.

2. Orbit Parameter: C

The Jacobi constant C of the targeted orbit is used in this paper to specify which orbit is being targeted within the family. There are numerous ways to identify a particular three-body orbit within its family [28,41]. The Jacobi constant is used in this paper because it also provides information about the corresponding forbidden regions and allowable motion of spacecraft with that Jacobi constant [1].

3. Sun–Earth–Moon Angle: θ

The parameter θ is defined to be the angle between the sun–Earth line and the Earth–moon line. It is a required parameter needed to convert between the two three-body systems in the patched three-body model. Figure 21 shows an example of the geometry and the definition of θ.

4. Arrival Location: τ

Each point on a periodic orbit may be uniquely described by the parameter τ, a parameter analogous to a conic orbit's mean anomaly. In this paper, τ ranges from 0 to 1, representing a revolution number rather than an angle [1]. Figure 22 shows a plot of the definition of τ when applied to two halo orbits.

5. Perturbation Direction: p

To construct a trajectory in the stable invariant manifold of a given unstable orbit, one takes the state of the orbit at a given τ value and perturbs that state along the direction of the stable eigenvector [1,29]. The perturbation may occur in two directions: an interior or an exterior direction, as illustrated in Fig. 2. In this study, the parameter p is a discrete variable that may be set to interior or exterior, indicating the direction of the perturbation.

6. Manifold Propagation Time: Δt_m

The trajectory in the given three-body orbit's stable manifold is propagated backward in time for an amount of time equal to Δt_m. Typically when propagated backward in time, the trajectories that lead to desirable BLTs depart the vicinity of the moon, traverse their apogee, fall toward the Earth, and then intersect a desirable altitude above the surface of the Earth. However, BLTs may also be constructed that pass near the Earth once or several times before intersecting the desirable altitude above the surface of the Earth. Such trajectories must be propagated long enough to allow the desirable perigee passage to occur. Thus, the parameter Δt_m is important to ensure that the proper perigee passage is being implemented by the BLT.

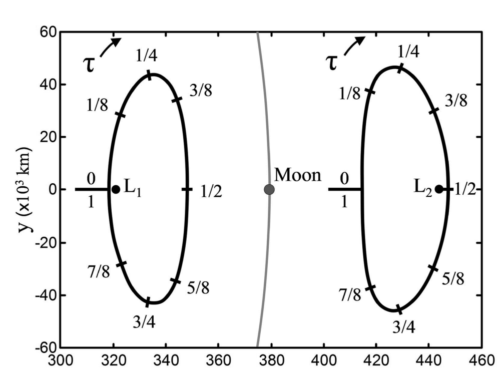

Fig. 22 Two halo orbits shown demonstrate how the parameter τ moves from 0 to 1 about an orbit.

Table 1 **Summary of the six parameters used to produce BLTs in the patched three-body model**

Parameter	Domain	Description
F	Discrete	Target three-body orbit family
C	Continuous	Jacobi constant of target orbit
θ	Continuous (0–360 deg)	Sun–Earth–moon angle
τ	Continuous (0–1)	Arrival location on the target orbit
p	Discrete	Perturbation direction
Δt_m	Continuous	Propagation duration

7. Discussion Regarding Parameters

The set of parameters used in this paper does not contain all continuous variables as other sets of orbital elements do, such as the Keplerian orbital element set of a two-body orbit. The present parameter set also requires knowledge about how to use it (e.g., how to build the target lunar orbit given the parameters F and C). Nonetheless, this set may be used to uniquely describe any low-energy ballistic transfer between the Earth and an unstable lunar three-body orbit. Table 1 summarizes the parameter set.

B. Producing the BLT

The process of producing a BLT given the parameter set F, C, θ, τ, p, and Δt_m is very simple and will be described henceforth.

1) First of all, one must build the target Earth–moon orbit. The desired orbit must be unstable and may be identified using the parameters F and C, as already defined. The example BLT presented in this paper has been produced using an orbit in the family F of southern halo orbits about the Earth–moon L_2 point. The specific orbit has been identified in its family by the value of C, equal to 3.05.

2) The parameter θ specifies the location of the moon, and hence the target orbit, with respect to the Earth and sun in the patched three-body model. The example BLT used an initial θ value of approximately 293.75 deg. This may be verified by inspecting the final location of the moon in Figs. 11–14. Because the BLT is produced backward in time, the value of θ specifies the *final* position of the moon.

3) The parameter τ specifies a particular state in the unstable three-body orbit. The example BLT implemented a τ value of approximately 0.74, corresponding to a point roughly three quarters around the orbit from the orbit's reference point (the point at which the orbit crosses the x axis with positive $\dot{y}$) [1,41].

4) The particular state in the target orbit is then perturbed to construct a single trajectory in the stable manifold of the orbit. The magnitude of this perturbation is given by ϵ; the direction is given by the orbit's monodromy matrix [42] and the parameter p. The orbit's monodromy matrix is used to compute the orbit's stable and unstable eigenvectors; the stable eigenvector is then mapped to the given τ value using the orbit's state transition matrix [1,29]. The example BLT implemented a trajectory in the halo orbit's *exterior* manifold with the value of ϵ set proportional to a 100 km perturbation.

5) The resulting state is then used as the initial condition to construct a trajectory in the stable manifold of the three-body orbit. This trajectory is propagated backward in time for a duration of time equal to Δt_m. The trajectory that produced the example BLT has been propagated for approximately 28.53 nondimensional Earth–moon time units (approximately 123.9 days) before encountering the desired perigee point (i.e., the desired LEO injection point).

6) The final step in the construction of a BLT is to connect this trajectory with a prescribed LEO parking orbit or with the surface of the Earth. It is unlikely that an arbitrary set of parameters will yield a BLT that connects with its prescribed LEO starting conditions. In such a case, either the parameters should be adjusted [1] or a bridge must be constructed to connect the spacecraft's origin with the BLT [2]. This study is beyond the scope of this paper.

C. Discussion

The parameter set derived here is very useful if a mission designer needs to build a transfer to a specific lunar orbit that cannot exceed some maximum transfer time. In that case, the parameters F, C, and Δt_m are fixed. By setting Δt_m to the maximum transfer duration, one ensures that no BLTs are constructed that require excessive transfer time, but one still permits BLTs that require less transfer time. The three remaining parameters are conveniently well-defined. The parameter p is binary and the parameters θ and τ are cyclic. Thus, mission designers can explore all possible BLTs to a target orbit by producing two maps: one map of θ vs τ with p set to *exterior* and another identical map with p set to *interior*. The exploration of these maps is outside of the scope of this paper.

Other methods have been described in the literature that also describe parameter sets to target BLTs. The majority of these methods start with a spacecraft in orbit about the Earth and target a maneuver for that spacecraft to perform to reach the moon's vicinity via a BLT. For instance, Belbruno and Carrico [8] developed a set of parameters that describe the six-dimensional state that a spacecraft would need to obtain to reach the moon's vicinity via a low-energy transfer. Five parameters are specified, including an epoch t, the spacecraft's radial distance from Earth (r_E), its longitude α_E, its latitude δ_E, and its flight-path azimuth σ_E. Then the spacecraft's speed V_E and flight-path angle γ_E are varied to target a prescribed radial distance from the Earth (r_M) and a prescribed inclination i_M, which would ultimately send the spacecraft in the general direction of a low-energy transfer. The advantage of this method is that the spacecraft's initial orbit at the Earth is well-defined, which is useful when a transfer must be designed for a spacecraft that is already in orbit about the Earth. However, the technique requires a great deal of predetermined knowledge of the problem, including a priori estimates for the values of r_M, i_M, V_E, γ_E, and t (t is specified to obtain a proper sun–Earth–moon angle). The procedure is therefore constrained to build a transfer with a predefined geometry that may not be ideal.

Operationally, it is likely that a combination of these two approaches works the best to produce practical BLTs. A BLT may then be constructed that starts from a prescribed orbit, ends at a

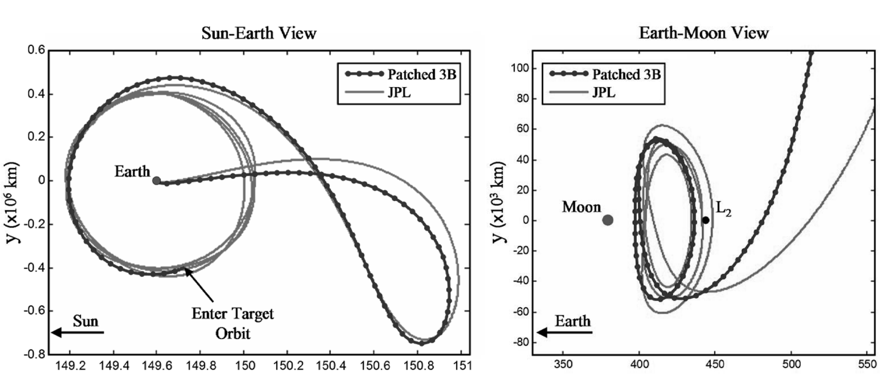

Fig. 23 Two plots comparing the general characteristics of the patched three-body BLT (dotted line) and the JPL DE405 BLT (solid line). The trajectories are viewed from above in the sun–Earth (left) and Earth–moon (right) synodic reference frames.

Table 2 Comparison of certain parameters of the LEO parking orbit and departure conditions for the BLTs produced in the patched three-body and JPL DE405 models

Parameter	Patched three-body BLT	JPL DE405 BLT
LEO parking orbit		
Altitude	185 km	185 km
Ecliptic inclination	51.249 deg	54.387 deg
Velocity	7.793 km/s	7.793 km/s
Parameters of the departure asymptote with respect to the ecliptic[a]		
Right ascension	8.019 deg	13.119 deg
Declination	1.654 deg	4.154 deg
Additional orbital elements of the BLT departure with respect to the Earth		
Specific energy	−0.366053 J/kg	−0.349478 J/kg
Eccentricity	0.987947	0.988494
Semimajor axis	544,503.0 km	570,328.1 km
Insertion ΔV		
Insertion ΔV	3.263928 km/s	3.196411 km/s

[a] The departure asymptote is defined here to be the state of the spacecraft approximately 200,000 km from the Earth.

specified lunar orbit, and most likely includes one or two small trajectory correction maneuvers to connect the segments.

VII. Validating BLTs in the JPL DE405 Model

The BLT that has been analyzed in this paper has been constructed in the patched three-body model to take advantage of all dynamical systems tools that exist in the three-body system. The patched three-body model is certainly useful for modeling and analyzing theoretical BLTs, but BLTs need to be constructed in more realistic models of the solar system to be useful for real missions. The BLT presented here has been converted into the JPL DE405 planetary and lunar ephemeris model of the solar system [43] using a multiple-shooting differential corrector [1,18,39]. This section compares the results of the conversion to discuss the validity of the patched three-body approximation.

Figure 23 shows the patched three-body BLT and the JPL DE405 BLT plotted on the same axes. The trajectories are shown from above in the sun–Earth and Earth–moon synodic reference frames. One can see that the general characteristics of the trajectories are the same. One slight difference is that the two trajectories depart the Earth at different angles and therefore pass by the moon slightly differently. In addition, the moon's orbit is not circular in the JPL DE405 reference frame, which helps to explain the variations in the JPL DE405 trajectory as it approaches and traverses the final Earth–moon three-body orbit.

Table 2 compares several performance parameters of the JPL DE405 BLT with the patched three-body BLT. One can see that both BLTs depart the Earth from 185 km circular LEO parking orbits. The ecliptic inclination of each parking orbit is approximately the same. The right-ascension and declination values of the departure asymptote of each trajectory with respect to the ecliptic are only slightly different in each model. To be consistent between models, these values are computed at the point at which the trajectories reach a distance of 200,000 km from the Earth. These parameters have not been included as constraints in the conversion process. Because the departure trajectory is somewhat different in each model, the trajectories each pass by the moon differently. Consequently, the two BLTs require slightly different ΔV magnitudes at the injection point from their corresponding LEO parking orbits. The ΔV magnitudes, as well as other orbital elements with respect to the Earth at the injection point, are compared in the table.

VIII. Comparison with Conventional Transfers

The example BLT produced in this paper requires a total ΔV of approximately 3.196 km/s to transfer a spacecraft from a 185 km LEO parking orbit to a quasi-halo orbit about the moon's L_2 point in the JPL DE405 model. In contrast, a conventional 5-day direct transfer between a 185 km LEO parking orbit and this same lunar halo orbit requires two large maneuvers and a total ΔV of approximately 3.980 km/s [1,2]. The BLT requires three months more transfer time, but approximately 19.7% less ΔV. Furthermore, the entire ΔV budget of the BLT presented here (except for navigation and other operational maneuvers) is executed in a single maneuver, which enables a simplified spacecraft system compared with a conventional transfer.

IX. Conclusions

This paper has studied how to analyze, model, and construct low-energy ballistic lunar transfers (BLTs) using dynamical systems techniques in the patched three-body model.

It has been shown that dynamical systems methods allow mission designers to compartmentalize the design of a BLT. In this way, one may construct the transfer from the Earth to the lunar three-body staging orbit independently from the transfer away from that orbit. A spacecraft may depart the lunar three-body orbit on a transfer to another three-body orbit, to a low lunar orbit, to the surface of the moon, or it may depart on a return trajectory to the Earth.

This paper has analyzed an example BLT from an energy perspective. It has been found that the sun's gravity effectively raises the two-body specific energy of a spacecraft with respect to the Earth during the early portion of the BLT. Similarly, it has been shown that a spacecraft's two-body specific energy with respect to the moon drops below zero as the spacecraft approaches the lunar halo orbit. It has also been shown that the moon effectively changes a spacecraft's sun–Earth Jacobi constant and that the sun effectively changes a spacecraft's Earth–moon Jacobi constant. One may conclude that a spacecraft may change its Jacobi constant in one system for very little energy by transferring to the other three-body system and back again.

This paper has shown that a low-energy BLT may be used to transfer material to the moon using much less ΔV than if that material were transferred to the moon using conventional methods. The example BLT produced in this paper requires a transfer duration of about 100 days and a total ΔV of approximately 3.196 km/s, compared with a conventional 5-day transfer that requires approximately 3.98 km/s to the same three-body orbit. The BLT requires about 19.7% less ΔV than the conventional transfer. The result is that the same launch vehicle can send much more payload to a lunar three-body orbit using a BLT rather than a conventional transfer.

Acknowledgment

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D. Spencer
Associate Editor